Output feedback control for a class of nonlinear delayed systems

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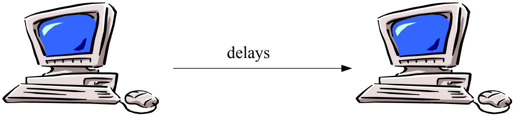
Problem formulation



Data transmission

- Networked systems ⇒ delays affecting :
  - output measurements
  - input
- Solution = output feedback design based on state prediction system

Outline

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Class of systems

Uniformly observable systems

$$\begin{array}{rcl} \dot{x} & = & Ax + \phi(x,u) \\ y & = & Cx(t-\tau) \end{array}$$

where $A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \dots & \dots & 1 \\ 0 & \dots & \dots & \dots & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix}$ and

$\phi(x,u) = \begin{pmatrix} \phi_1(x,u) \\ \vdots \\ \phi_n(x,u) \end{pmatrix}$

Hypotheses

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- The functions $\phi_i(x,u)$ are triangular in x, i.e $\frac{\partial \phi_i(x,u)}{\partial x_{k+1}} = 0$, for $k = i, \ldots, n-1$
- The functions $\phi_i(x,u)$ are globally Lipschitz, uniformly in u
- The time delay τ is supposed constant and known

Notation

$x_j(t) = x(t - \tau + j\frac{\tau}{m})$, for $j = 1, \ldots, m$

High gain observer design

Cascaded observers

$$
\begin{aligned}
\dot{\hat{x}}_1 &= A\hat{x}_1 + \phi(\hat{x}_1) - \theta\Delta^{-1}S^{-1}C'C(\hat{x}_1(t-\tfrac{\tau}{m}) - x(t-\tau)) \\
\hat{y}_1 &= C\hat{x}_1(t-\tfrac{\tau}{m}) \\
\vdots &= \vdots \\
\dot{\hat{x}}_j &= A\hat{x}_j + \phi(\hat{x}_j) - \theta\Delta^{-1}S^{-1}C'C(\hat{x}_j(t-\tfrac{\tau}{m}) - \hat{x}_{j-1}(t)) \\
\hat{y}_j &= C\hat{x}_j(t-\tfrac{\tau}{m}) = C\hat{x}_{j-1}(t)
\end{aligned}
$$

where

- $\theta > 1$
- S is a symmetric positive definite matrix, unique solution of the following algebraic Lyapunov equation

$$SA + A^T S - C^T C = -S$$

- $\Delta = Diag\left(1, \ldots, \frac{1}{\theta^{i-1}}, \ldots, \frac{1}{\theta^{n-1}}\right)$

$\Rightarrow$ exponential convergence $\hat{x}_j(t) \to x(t-\tau+j\frac{\tau}{m})$ and $\hat{x}_m(t) \to x(t)$

Sketch of the proof : case m=1

- case of a sufficiently small delay
- $\bar{x} = \Delta(\hat{x} - x)$
- $\dot{\bar{x}} = \theta(A - S^{-1}C^T C)\bar{x} + \Delta(\phi(\hat{x}, u) - \phi(x, u)) + \theta S^{-1}C' C \int_{t-\tau}^{t} \dot{\bar{x}}(s)ds$
- Lyapunov-Krasovskii functional $W = \bar{x}^T S\bar{x} + \int_{t-\tau_1}^{t} \int_{s}^{t} ||\dot{\bar{x}}(\xi)||^2 d\xi ds$
- Using the mean value theorem, the Lipschitz property of ϕ and a Jensen inequality, one obtains

$$\dot{W} + \frac{1}{\sqrt{\theta}} W \leq 0$$

provided that

$$\begin{cases} \theta \geq \max\{2, (k_1 + k_2 + \frac{1}{\sqrt{2}})\} \\ \tau_{max} = \frac{1}{\theta^2}. \end{cases}$$

where k_1 and k_2 are independent of θ

Cascaded high gain observers : step 1

- convergence step by step
- **Step 1 :** consider the first observer

$$\begin{aligned} \dot{\hat{x}}_1 &= A\hat{x}_1 + \phi(\hat{x}_1) - \theta\Delta^{-1}S^{-1}C^T C(\hat{x}_1(t-\tfrac{\tau}{m}) - x(t-\tau)) \\ \hat{y}_1 &= C\hat{x}_1(t-\tfrac{\tau}{m}) \end{aligned}$$

- delay to handle $= \frac{\tau}{m} \Rightarrow$ the previous result can be applied :

if $\frac{\tau}{m} \leq \frac{1}{\theta^2}$ i.e. if $m \geq \theta^2\tau$ then

$$\hat{x}_1(t) \to x_1(t) = x(t - \tau + \frac{\tau}{m}) = x(t - (m-1)\frac{\tau}{m})$$

Cascaded high gain observers : step j

- **Step j :** for j=2,. . . ,m, consider

$$\begin{aligned} \dot{\hat{x}}_j &= A\hat{x}_j + \phi(\hat{x}_j) - \theta\Delta^{-1}S^{-1}C^T C(\hat{x}_j(t - \tfrac{\tau}{m}) - \hat{x}_{j-1}(t)) \\ \hat{y}_j &= C\hat{x}_j(t - \tfrac{\tau}{m}) = C\hat{x}_{j-1}(t) \end{aligned}$$

- $\Rightarrow \hat{x}_j \rightarrow x(t - \tau + j\frac{\tau}{m})$

Numerical simulations

Example

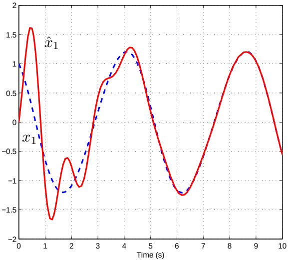
$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + 0.5\tanh(x_1(t) + x_2(t)) + x_1(t)u(t) \\ y(t) = x_1(t-\tau) \end{cases}$$

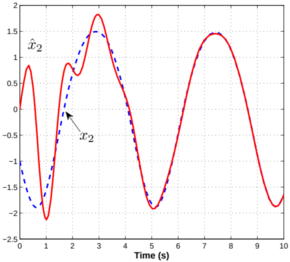
with $u(t) = 0.1\sin(0.1t)$, $\theta = 2$

$\Rightarrow$ 2 examples : $\tau = 0.25$s with 1 observer and $\tau = 1$s with 4 cascaded observers

Small delay : 1 observer

True and estimated states
$\tau = 0.25$s, $m = 1$

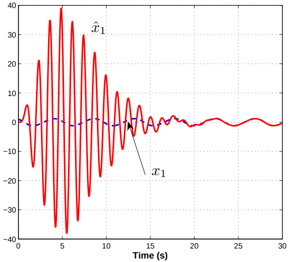


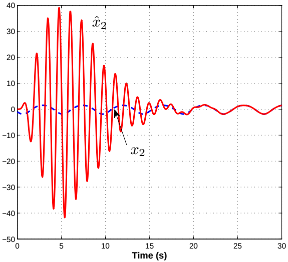


Larger delay : 4 observers

True and estimated states

$\tau = 1$s, $m = 4$





Outline

1. Observer design
2. Controller design
3. Conclusion

Output feedback control law

- Class of systems

$$\begin{cases} \dot{x}(t) = Ax(t) + \phi(x(t)) + bu(t-\tau) \\ y(t) = Cx(t) \end{cases}$$

- stabilizing feedback control :

$$u(t) = -\lambda^n b^T \bar{S} \Delta_\lambda \hat{x}(t+\tau)$$

where $\lambda > 0$ is a tuning parameter and Δ_λ is defined like Δ

- variable change : $z(t) = \hat{x}(t+\tau) \Leftrightarrow \hat{x}(t) = z(t-\tau)$
- then the observer is expressed as :

$$\dot{z}(t) = Az(t) + \phi(z(t)) + bu(t) - \theta\Delta_\theta^{-1} S^{-1} C^T \left(y(t) - z(t-\tau)\right)$$

where $u(t) = -\lambda^n b^T \bar{S} \Delta_\lambda z(t)$

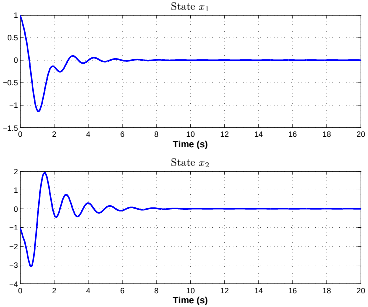
- the key point = to use the delayed control law $u(t-\tau)$ in the system dynamics (= the real applied control), whereas we use $u(t)$ in the observer's dynamics

$\Rightarrow$ we are brought back to the previous problem

Example (1/2)

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + 0.5\tanh(x_1(t) + x_2(t)) + u(t-\tau) \\ y(t) = x_1(t) \end{cases}$$

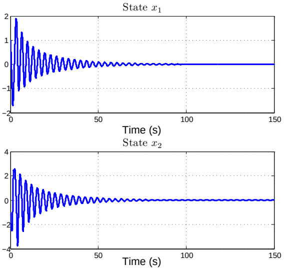
- Parameters : $\tau = 0.25s$, $\theta = 2$, $\lambda = 2$
- $m = 1$ observer
- Initial conditions :
  $x(t) = \begin{pmatrix} 1 & -1 \end{pmatrix}^T$,
  $\hat{x}(t) = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$, $\forall t \in [-\tau, 0]$



Example (2/2)

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + 0.5\tanh(x_1(t) + x_2(t)) + u(t-\tau) \\ y(t) = x_1(t) \end{cases}$$

- Parameters : $\tau = 0.5s$, $\theta = 2$, $\lambda = 2$
- $m = 2$ observers
- Initial conditions :
  $x(t) = \begin{pmatrix} 1 & -1 \end{pmatrix}^T$,
  $\hat{x}(t) = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$, $\forall t \in [-\tau, 0]$



Outline

Conclusion

Summary

- new predictor based on high gain observer
- can be applied to the class of nonlinear uniformly observable systems to cope with
  - input delays
  - output delays

  arising from communication networks for example

Future work

- handling variable delays
- design of adaptive observers for nonlinear systems with delayed output and uncertain or unknown parameters